Mechanical properties of fluids

Session – 02

Fluid Dynamics

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Fluid dynamics

- Types of flow
- Streamlines
- Equation of continuity
- Bernoulli's theorem
- Venturimeter
- Velocity of efflux
- Daily life examples

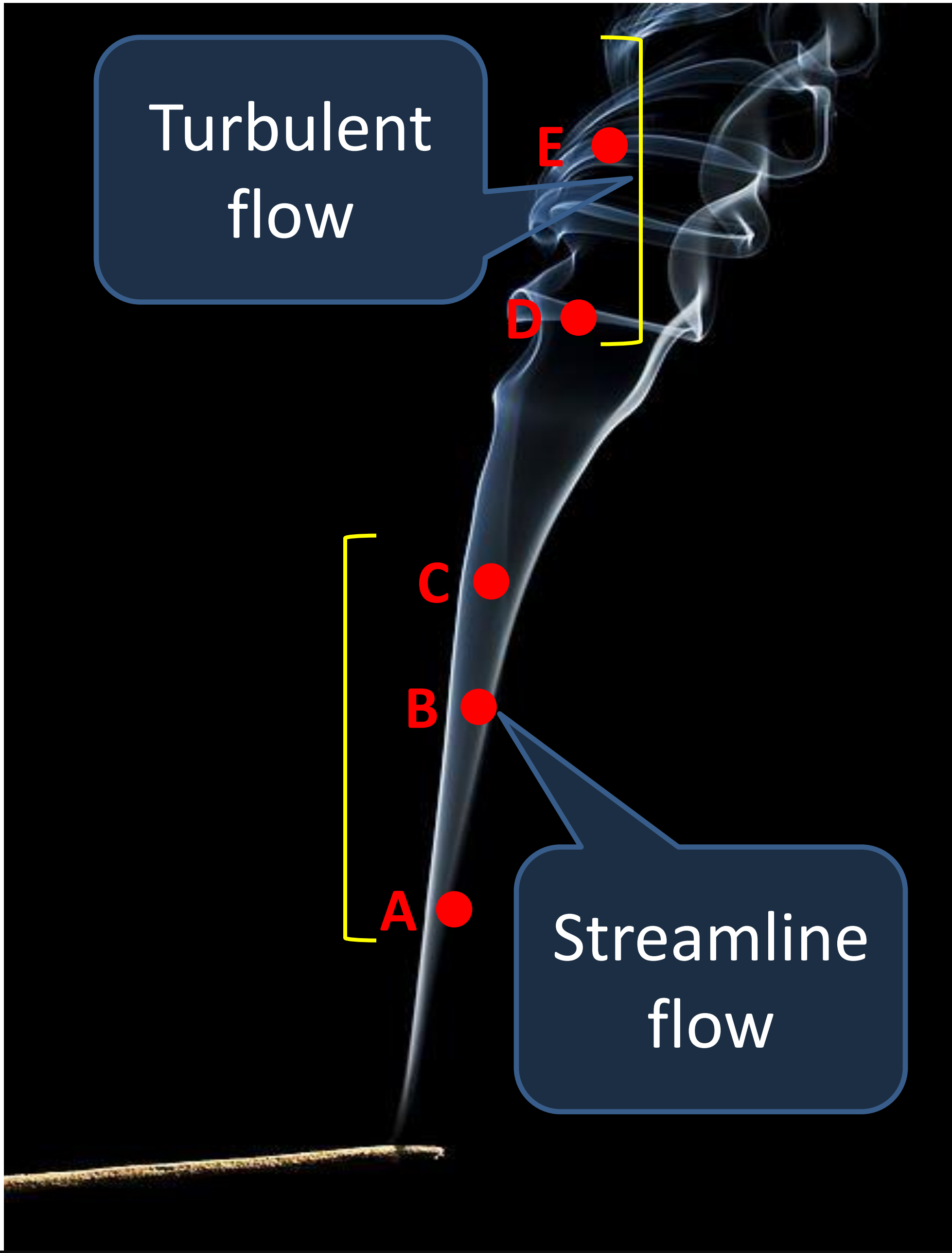
Streamlined flow

Flow of a fluid is said to be streamlined if all the particles passing through a set of points have the same set of velocities.

Turbulent flow

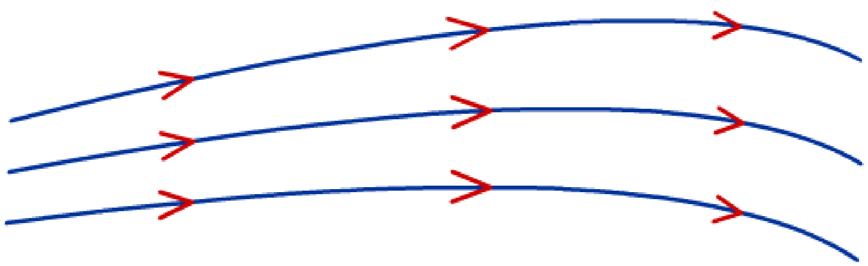
Flow of a fluid is said to be turbulent if all the particles passing through a set of points do not have the same set of velocities.

Video link



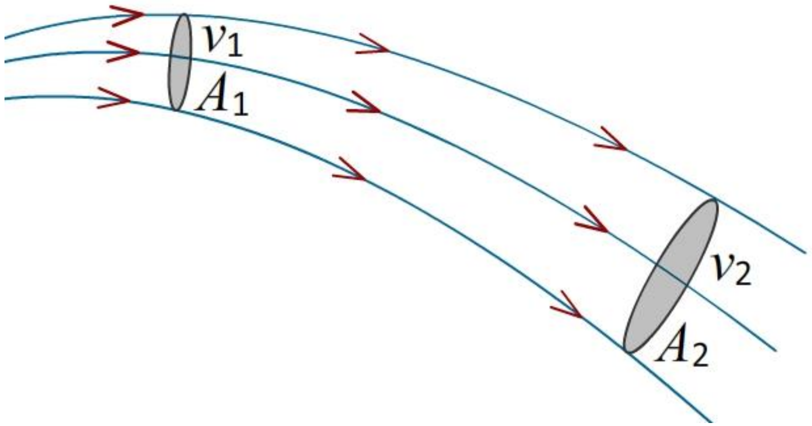
Properties of stream lines

- Streamline is an imaginary line drawn along the path followed by a particle of the fluid
- Streamlines are dense where the velocity of fluid is high
- Streamlines are sparse where the velocity of fluid is low
- Two streamlines do not intersect each other
- Tangent drawn to a streamline at any point gives the direction of flow of fluid at that point



Equation of continuity

For an incompressible, non-viscous , irrotational fluid in a streamlined flow the product of area of cross-section (A) and velocity (v) is constant at any point of its flow.



Let A_1 and A_2 be the areas of cross-section at the two ends of the tube and v_1 and v_2 be velocities of the fluid at these ends respectively.

In a small interval of time (dt) the amount of fluid entering the tube at A_1 is given by

$$dm_1 = \rho A_1 v_1 dt \quad \text{— i}$$

In the same interval of time the amount of fluid leaving the tube at A_2 is given by

$$dm_2 = \rho A_2 v_2 dt \quad \text{— ii}$$

Using law of conservation of mass and considering the incompressible nature of fluid, we get

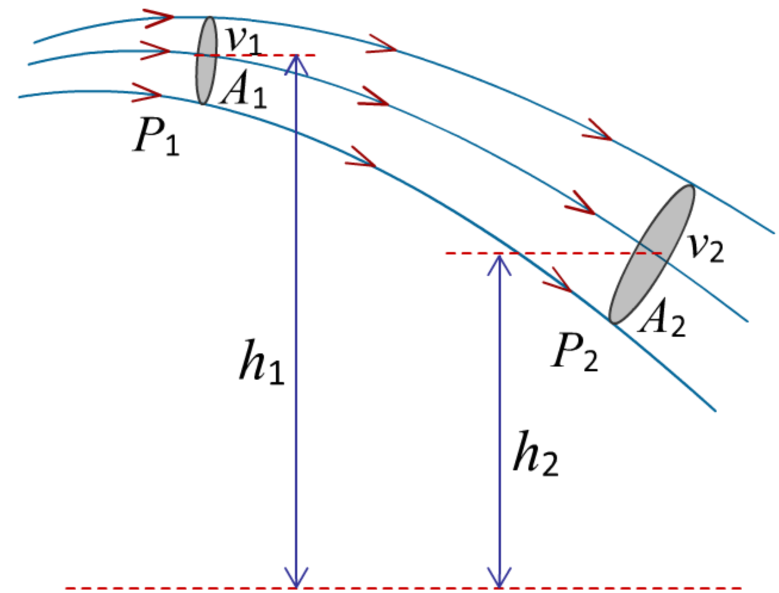
$$dm_1 = dm_2$$

$$\rho A_1 v_1 dt = \rho A_2 v_2 dt$$

$$\boxed{A_1 v_1 = A_2 v_2}$$

Bernoulli's principle

For a non-viscous, irrotational, incompressible fluid in stream line flow, the sum of pressure energy, kinetic energy and gravitational potential energy per unit volume is constant.



Let A_1 and A_2 be areas of cross-section, v_1 and v_2 be velocities, h_1 and h_2 be the heights and P_1 and P_2 be the pressures at these ends respectively.

In a certain interval of time volume of fluid entering the tube is equal to the volume leaving the tube (V)

Work done on the fluid as it enters the tube is given by

$$W_1 = P_1 V \quad \text{i}$$

Work done on the fluid as it leaves the tube is given by

$$W_2 = -P_2 V \quad \text{ii}$$

Total work done on the fluid as it passes through the tube is given by

$$W = (P_1 - P_2)V \quad \text{iii}$$

Initial gravitational potential energy is

$$PE_1 = \rho V g h_1 \quad \text{iv}$$

Final gravitational potential energy is

$$PE_2 = \rho V g h_2 \quad \text{v}$$

Change in gravitational potential energy is

$$\Delta PE = \rho V g (h_2 - h_1) \quad \text{vi}$$

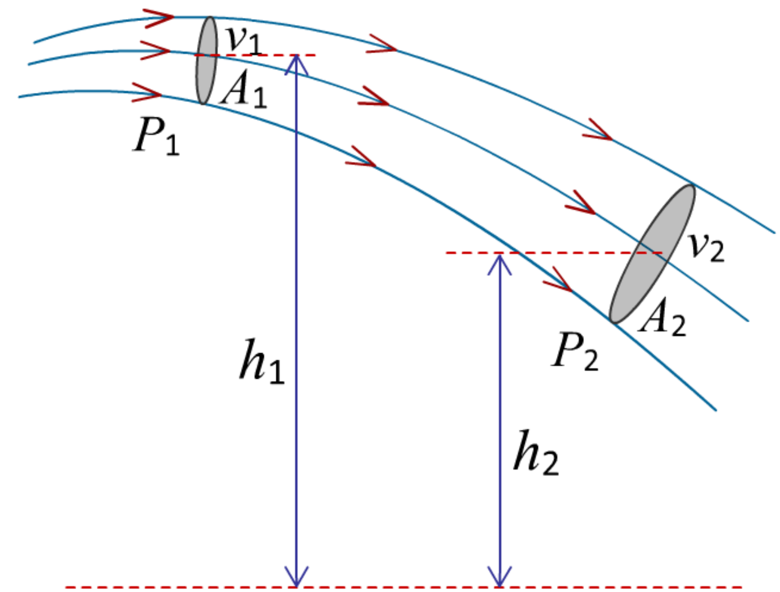
Change in kinetic energy is

$$\mathrm{d}KE = \frac{1}{2}\rho V\left(v_2^{\ 2} - v_1^{\ 2}\right) \quad \text{vii}$$

Using work energy principle

$$W = \Delta PE + \Delta KE$$

$$(P_1 - P_2)V = \rho V g (h_2 - h_1) + \frac{1}{2}\rho V\left(v_2^{\ 2} - v_1^{\ 2}\right)$$



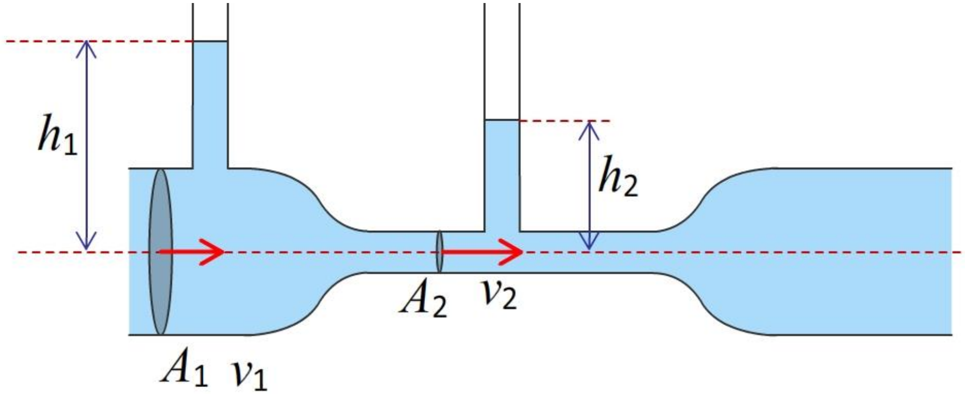
For unit volume of fluid we get

$$(P_1 - P_2) = \rho g (h_2 - h_1) + \frac{1}{2}\rho\left(v_2^{\ 2} - v_1^{\ 2}\right)$$

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^{\ 2} = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^{\ 2}$$

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}}$$

Venturimeter (for liquids)



It is a device used to measure speed of a fluid in a tube. It works on the basis of Bernoulli's principle.

Consider a streamlined flow of liquid through a venturimeter as shown in the figure.

Using Bernoulli's principle we get

$$P_1 + \rho g h + \frac{1}{2}\rho v_1^{\,2} = P_2 + \rho g h + \frac{1}{2}\rho v_2^{\,2}$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^{\,2} - \frac{1}{2}\rho v_1^{\,2}$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^{\,2} - v_1^{\,2}\right) \quad \text{— i}$$

Pressure difference is given by

$$P_1 - P_2 = \rho g\left(h_1 - h_2\right) \quad \text{— ii}$$

Using equation of continuity we get

$$v_2 = \frac{A_1}{A_2} v_1 \quad \text{— iii}$$

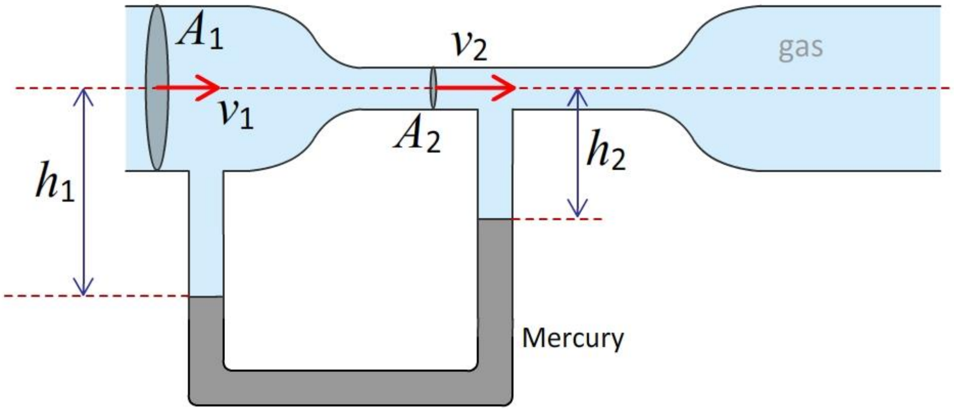
Substituting eqs (ii) and (iii) in (i) we get

$$\rho g\left(h_1 - h_2\right) = \frac{1}{2}\rho\left(\frac{A_1^{\,2}}{A_2^{\,2}} v_1^{\,2} - v_1^{\,2}\right)$$

$$v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1^{\,2}}{A_2^{\,2}} - 1\right)}}$$

Venturimeter (for gases)

In this case mercury is used as the liquid in the manometer to measure the pressure difference.



Consider a streamlined flow of gas through a venturimeter as shown in the figure.

Using Bernoulli's principle we get

$$P_1 + \rho gh + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \text{ — } \boxed{\text{i}}$$

Pressure difference is given by

$$P_1 - P_2 = \rho g\left(h_1 - h_2\right) \text{ — } \boxed{\text{ii}}$$

Using equation of continuity we get

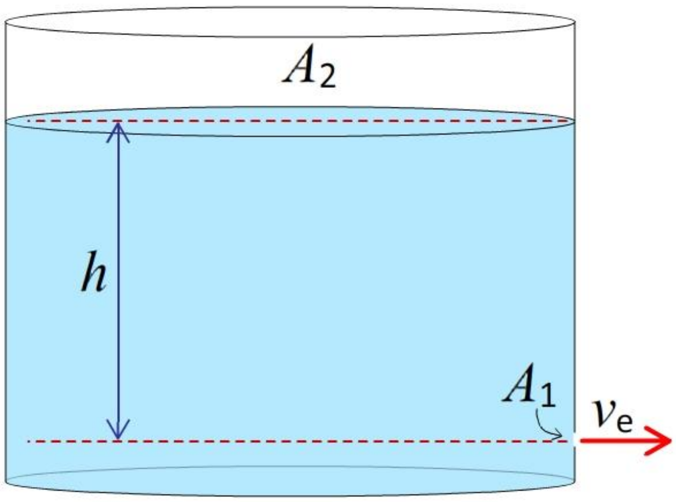
$$v_2 = \frac{A_1}{A_2}v_1 \text{ — } \boxed{\text{iii}}$$

Substituting (ii) and (iii) in (i) we get

$$\rho g\left(h_1 - h_2\right) = \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2}v_1^2 - v_1^2\right)$$

$$v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1^2}{A_2^2} - 1\right)}}$$

Velocity of efflux



Torricelli's theorem: Velocity of efflux of liquid from a narrow aperture, is equal to the velocity of a freely falling body (when $A_2 >> A_1$).

Consider a cylindrical container with a small aperture near its base. Let A_2 and A_1 be areas of top surface and aperture respectively. Height of top surface, from the aperture is h.

Using Bernoulli's principle we get

$$P_o + \rho g h + \frac{1}{2}\rho v_2^2 = P_o + \rho g 0 + \frac{1}{2}\rho v_1^2$$

$$\frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2 + \rho g h$$

$$\frac{1}{2}\left(v_1^2 - v_2^2\right) = gh$$

$$v_1^2 - v_2^2 = 2gh \qquad \text{i}$$

Using equation of continuity we get

$$v_2 = \frac{A_1}{A_2} v_1$$

$$v_2 \approx 0 \qquad (\because A_2 >> A_1)$$

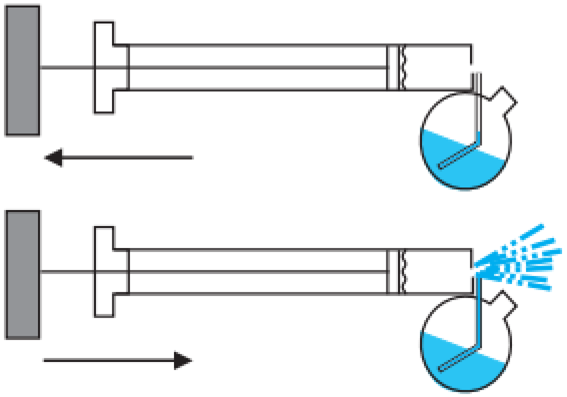
Substituting this in eq (i) we get

$$\boxed{v_e = \sqrt{2gh}}$$

Other (day to day) applications

Carburetor of automobile has a Venturi channel (nozzle) through which air flows with a high speed. The pressure is then lowered at the narrow neck and the petrol (gasoline) is sucked up in the chamber to provide the correct mixture of air to fuel necessary for combustion

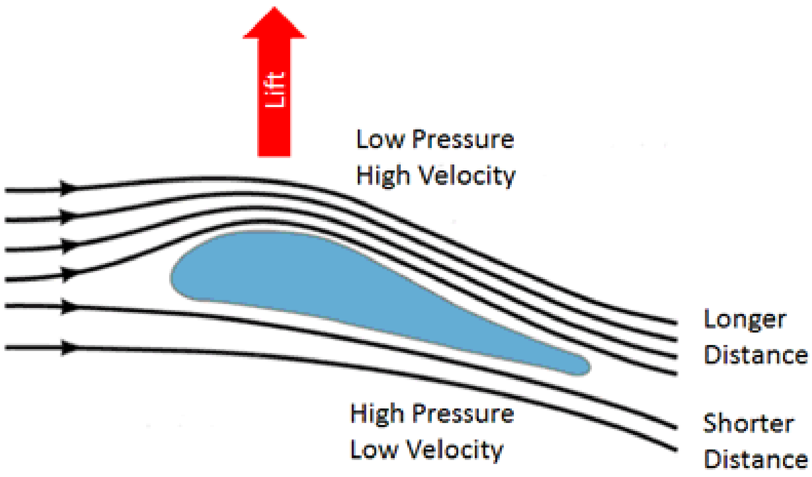
Filter pumps or aspirators, Bunsen burner, atomizers and sprayers used for perfumes or to spray insecticides work on the same principle.



Other (day to day) applications

Dynamic lift of an aircraft

Dynamic lift is the force that acts on a body, such as airplane wing, a hydrofoil or a spinning ball, by virtue of its motion through a fluid.



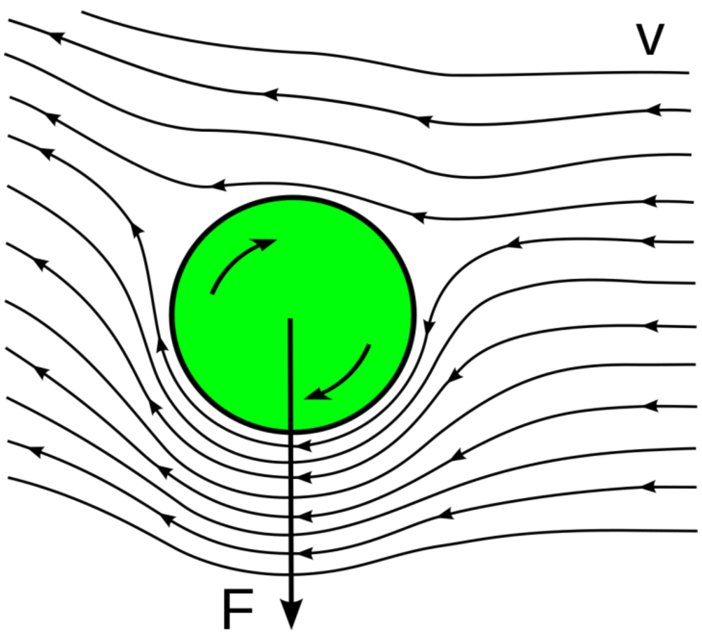
Aerofoil or lift on aircraft wing is a solid piece shaped to provide an upward dynamic lift when it moves horizontally through air. Cross-section of the wings of an aeroplane is similar to the aerofoil shown in the figure (with streamlines around it). When the aerofoil moves against the wind, the orientation of the wing relative to flow direction causes the streamlines to crowd together above the wing more than those below it. The flow speed on top is higher than that below it. There is an upward force resulting in a dynamic lift of the wings and this balances the weight of the plane.

Other (day to day) applications (Magnus effect)

Dynamic lift due to spinning is called ***Magnus effect***.
It results in deviation from its parabolic trajectory as it moves through air.

When a ball moves through a fluid without spin velocity of fluid (air) above and below the ball at corresponding points is the same resulting in zero pressure difference. Therefore there is no upward or downward force on the ball due to air.

A spinning ball drags air along with it. Velocity of air relative to the ball is backwards. Additionally due to the spin of the ball, velocity of air below the ball relative to the ball is larger than the velocity of air above it (as indicated by stream lines crowded below and rarified above the ball).
This difference in the velocities of air results in the pressure difference between the lower and upper faces and there is a net downward force on the ball.



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